Isotopes and Weighted Averages

We now know how to determine the numbers of protons, neutrons, and electrons in an atom and convey them using shorthand notation. But there is a problem with the periodic table. In the previous pages, we cherry-picked examples to teach you the concept but perhaps many of you have noticed that most of the elements on the periodic table are not whole numbers when you examine the masses. For instance:

3	20	56	17
Li	Ca	Ba	Cl
6.9	40.1	137.3	35.5

Why are some of these numbers not *whole* numbers? The bottom number represents the atomic mass which is the number of protons plus neutrons. So according to our previous theory, either we have to round to a whole number (in which case why bother putting the decimal to begin with in so many elements) or we will have a fraction of a particle:

Lithium 6.9:	3 protons	3.9 neutrons?
Calcium 40.1:	20 protons	20.1 neutrons?
Barium 137.3:	56 protons	81.3 neutrons?
Chlorine 35.5:	17 protons	18.5 neutrons?

Can we have a fraction of a neutron? No, so what's the deal?

Isotopes

It turns out that elements are not quite as simple as advertised. It turns out there can be many different kinds of the same element. For instance, carbon exists naturally in two forms. Carbon with a mass of 12 and carbon with a mass of 14. Oxygen exists naturally as a mass of 15 and a mass of 16. These different forms of the same element are called ***isotopes***. Isotopes are *different forms of the same element with different numbers of neutrons but the same number of protons.*

For example:

Carbon-12	6 protons, 6 electrons, 6 neutrons
Carbon-14	6 protons, 6 electrons, ***8 neutrons***
Oxygen-16	8 protons, 8 electrons, 8 neutrons
Oxygen-15	8 protons, 8 electrons, ***7 neutrons***

Notice that for each isotope, the protons and electrons are the same (carbon MUST have 6 protons and oxygen MUST have 8 protons) but the neutrons are different so they have different masses.

Examine the charts below for all of the isotopes of the given element:

Uranium has three isotopes that are common:

Name	Atomic #	Atomic mass	protons	electrons	neutrons	
Uranium-234	92	234	92	92	142	$^{234}_{92}U$
Uranium-235	92	235	92	92	143	$^{235}_{92}U$
Uranium-238	92	238	92	92	146	$^{238}_{92}U$

Tin has ten isotopes that are common:

Name	Atomic #	Atomic mass	protons	electrons	neutrons	
Tin-112	50	112	50	50	62	$^{112}_{50}Sn$
Tin-114	50	114	50	50	64	$^{114}_{50}Sn$
Tin-115	50	115	50	50	65	$^{115}_{50}Sn$
Tin-116	50	116	50	50	66	$^{116}_{50}Sn$
Tin-117	50	117	50	50	67	$^{117}_{50}Sn$
Tin-118	50	118	50	50	68	$^{118}_{50}Sn$
Tin-119	50	119	50	50	69	$^{119}_{50}Sn$
Tin-120	50	120	50	50	70	$^{120}_{50}Sn$
Tin-122	50	122	50	50	72	$^{122}_{50}Sn$
Tin-124	50	124	50	50	74	$^{124}_{50}Sn$

But if there are different versions of the same element, which one do we choose? How do we get the mass that we put on the periodic table?

Weighted Averages

We have run into a conceptual problem. There are many different forms of various elements called isotopes. These isotopes are all the same element but have different numbers of neutrons so have different masses. Which one do we choose?

For example, boron has two common isotopes: boron-10 and boron-11. Which do we put on the periodic table? It turns out that we don't have to choose. If some of the atoms are boron 10 and some of the atoms are boron 11 then why don't we just average the two masses and put that number on the periodic table?

B-10 has a mass of 10
B-11 has a mass of 11

If we average them we get

$\frac{(10+11)}{2} = 10.5$ so…

5
B
10.8

Wait a minute! 10.8? Why doesn't the periodic table say 10.5? Clearly 10.5 is the average of 10 and 11 so where did 10.8 come from?

The answer lies in what, mathematically, an average really is. Let's examine the math we did above again:

$\frac{(10+11)}{2} = 10.5$ we could write it like this, though: $\frac{10}{2} + \frac{11}{2} = 10.5$

or we could do this:

$1/2\ (10) + 1/2\ (11) = 10.5$

which is really this:

$0.5\ (10) + (0.5)\ (11) = 10.5$

written in percentages it is:

$50\%\ (10) + 50\%\ (11) = 10.5$

So what are we really saying here? We're saying that half or 50% of each of the two numbers added together gets us our average. We're saying that the 10 and the 11 are of *equal importance.* But what if they aren't of equal importance? What if one of the numbers is *more* important than the other number? How could that be? What could make one of the isotopes of boron more important than the other?

Natural Abundance

The reason that the isotopes don't have the same importance is that they don't occur in nature equally. In fact, for most elements, one of the isotopes is the *vast* majority of the occurrence while the other isotopes are very rare. Let's examine boron again. It turns out:

B-10:	20% of all boron atoms are boron-10
B-11:	80% of all boron atoms are boron-11

Should these two atoms be treated equally when boron-11 atoms are four times more common than boron 10 atoms?

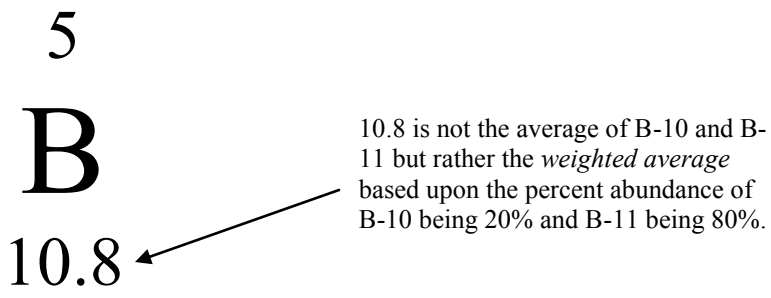
No, they shouldn't be so we have to adjust our math to account for these abundances. Our previous equation looked like this:

$$50\% (10) + 50\% (11) = 10.5$$

But that assumed that boron-10 and boron-11 were equally important. They are not as indicated by the percentages above. Therefore they must be *weighted* differently. Substituting those values in instead of the 50% values we see we get:

$$20\% (10) + 80\% (11) =$$
$$2 + 8.8 =$$
$$10.8$$

Wait a minute! 10.8? Why does that sound familiar? Remember this?



Oh yes, I remember now. 10.8 was the number on the periodic table that we couldn't figure out before. The average of 10 and 11 is 10.5 but the weighted average of B-10 and B-11 taking into account the fact that only 20% of all Boron atoms are B-10 and 80% of all Boron atoms are B-11 leads us to the weighted average of 10.8.

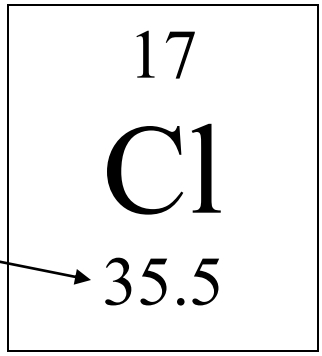
(% isotope 1)(mass isotope 1) + (% isotope 2)(mass isotope 2) = Weighted average

Examples:

A) Calculate the weighted average of chlorine atoms if of all the chlorine atoms in the world, 75% are Cl-35 and 25% are Cl-37.

(75%)(35) + (25%)(37) =
(0.75)(35) + (0.25)(37) =
26.25 + 9.25 = **35.5**

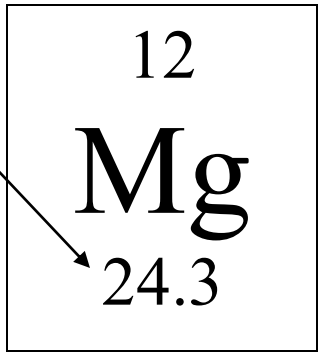
Checking the periodic table we see:



B) Calculate the weighted average of magnesium atoms if of all the magnesium atoms in the world, 79% are Mg-24, 10% are Mg-25, and 11% are Mg-26.

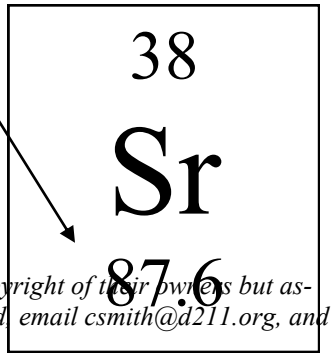
(79%)(24) + (10%)(25) + (11%)(26) =
(0.79)(24) + (0.10)(25) + (0.11)(26) =
18.96 + 2.5 + 2.86 = **24.3**

Checking the periodic table we see:



C) Knowing that strontium has a weighted average on the periodic table of 87.6, which of the following isotopes would you expect to be most abundant? Sr-84, Sr-86, Sr-87, or Sr-88? Explain why?

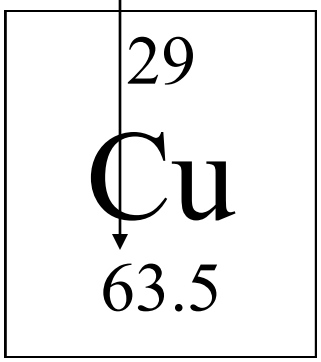
Strontium-88 should be the most abundant as the weighted average is closest to that value. The periodic table has its mass as 87.6 so the closest value is usually the most abundant.



More Examples:

A) Copper has two prominent isotopes, Cu-63 and Cu-65. What is the % abundance of each isotope?

Knowing from the periodic table that copper has a weighted average of 63.5, we can fill in our formula:



wtd avg = (% iso 1)(mass iso 1) + (% iso 2)(mass iso 2)

63.5 = (% iso 1)(63) + (% iso 2)(65)

These are the masses of the isotopes from above.

But what to do about the percentages? We only have one equation so we can't put in *x and y* as variables because there would be no way to solve for it. But, if we use a little logic, we can solve it using only one variable. For instance, let's just say that Cu-63 was 90%. What would Cu-65 be? 10%, of course. If Cu-65 was 60% then Cu-63 would have to be 40% because the percentages have to add up to 100% or "1" in decimal form. Thus:

63.5 = (x)(63) + (1-x)(65) so: 63.5 = 63x + 65—65x combining terms:

X is going to be the percentage of Cu-63 and the rest will be for Cu-65. We have to use "1" instead of "100" for the mast to work out in decimal form.

-1.5 = -2x x = 0.75 or 75%

Since x was assigned to Cu-63, it is 75%
That means Cu-65 is 25%

B) There are two common isotopes of lithium, Li-6 and Li-7. Calculate the % abundance of each isotope.

6.9 = (x)(6) + (1-x)(7) 6.9 = 6x + 7—7x -0.1 = -x x = 0.1 or 10%

Li-6 is 10% and Li-7 is 90%

C) There are two common isotopes of nickel, Ni-58 and Ni-60. Calculate the % abundance of each isotope.

58.7 = (x)(58) + (1-x)(60) 58.7 = 58x + 60—60x -1.3 = -2x x = 0.65

Ni-58 is 65% and Ni-60 is 35%

Questions

1. What is an isotope?
2. An isotope has the same number of _____ but different _____.
3. What is the difference between a normal average and a weighted average?
4. Why don't we use a normal mean to average all of the isotopes of each element?
5. Neon has two stable isotopes; Ne-20 which is 90% and Ne-22 which is 10%. Calculate the average mass of a neon atom.
6. Sulfur has two stable isotopes; S-32 which is 95% and S-34 which is 5%. Calculate the average mass of a sulfur atom.
7. Titanium's isotopes and abundances are listed below. Calculate the average mass of a titanium atom:

Ti– 46	8%
Ti-47	7.3%
Ti-48	73.8%
Ti-49	5.5%
Ti-50	5.4%

8. Potassium has two stable isotopes, K-39 and K-41. Which one should be more abundant in nature? Explain why.
9. Argon has two major isotopes; Ar-36 and Ar-40. Calculate the % abundance of each.
10. Silicon has two major isotopes; Si-28 and Si-29. Calculate the % abundance of each.